IB Maths AA HL Paper 1 Question Bank

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Section A

1. A fair die is thrown 3 times. Let X be the number of throws resulting in a six.

(a) Write down the probability mass function of X.

The probability mass function of X is given by the binomial probability distribution. In this case, the number of trials is 3, and the probability of success in each trial is 1/6, since it is a fair die. So the probability mass function of X is:

$P(X = x) = (3 \text{ choose } x) * (1/6)^{x} * (5/6)^{(3-x)}$ for x = 0, 1, 2, 3

(b) Find the expected value of X.

To find the expected value of X, we use the formula $E(X) = \sum xP(X = x)$ for x = 0, 1, 2, 3

$E(X) = (0 * (3 \text{ choose } 0) * (1/6)^{0} * (5/6)^{3}) + (1 * (3 \text{ choose } 1) * (1/6)^{1} * (5/6)^{2}) + (2 * (3 \text{ choose } 2) * (1/6)^{2} * (5/6)^{1}) + (3 * (3 \text{ choose } 3) * (1/6)^{3} * (5/6)^{0})$

$E(X) = 3/2$

2. Given that = 2 – i, *z*, find *z* in the form *a* + i*b*.

$z = (2 - i)(z + 2)$

$= 2z + 4 - iz - 2i$

$z(1 - i) = -4 + 2i$

$z = -3 - i$

3. Consider the cubic function $f(x) = ax^3 + 2x^2 + 3x + 4$. Find the value of a in each of the following cases.

a) the graph of the function passes through the point (1,10).

f(1)= 10, a=1

b) f(x) is divisible by (x-1)

f(1) = 0, a= -9

c) when f(x) is divided by (x-1), the remainder in the 10

f(1) = 10, a=1

4. Solve the equations

a) logx + log(x+1) = log6

x(x+1)=6

$x^2+x-6=0$

x=2

b) logx + log(x+3) = 10

x(x+3)=10

$x^2+3x-10=0$

x=2

c) log(x+18) - logx = 1

(x+18)/x= 1

4. Solve the equation $\tan^2 x = 3$ for $-\pi \leq x \leq \pi$

$\tan x = \pm\sqrt{3}$

For $\tan x = \sqrt{3}$

$x=\pi/3$, $x= -2\pi/3$

For $\tan x = -\sqrt{3}$

$x=-\pi/3$, $x= 2\pi/3$

5. Solve the following system of equations

$X + 3y - 2z = -6$

$2x + y + 3z = 7$

$3x - y + z = 6$

Back substitution gives $x = 1$, $y = -1$, $z = 2$.

6. A fair six-sided die, with sides numbered 1, 1, 2,3, 4, 5 is thrown. Find the mean and variance of the score.

Mean = $\frac{1}{6}(1+1+2+3+4+5) = 8/3$

Variance= $\frac{1}{6}(1+1+4+9+16+25) - 64/9 = 20/9$

7. Let $f(z) = z^2 - 8z + 20$

a) Find the discriminant of the quadratic function f

Discriminant= -16

b) Find the complex roots of equation $f(z) = 0$ in the form $z = x \pm yi$

$z= (8\pm4i)/2 = 4\pm2i$

c) Use factorisation to express f in the form of $f(z) = (z-h)^2+k$

$(z-4-2i)(z-4-2i)$

$= (z-4)^2+4$

8. Given the function $f(x) = x^2 - 3bx + (c+2)$, determine the values of b and c such as $f(1) = 0$ and $f'(3) = 0$.

$f(1) = 1 - 3b + c + 2 = 0$

$f'(x) = 2x - 3b$

$f'(3) = 6 - 3b = 0$

$b = 2$

$1 - 3(2) + c + 2 = 0$

$c =3$

9. If $\ln(2x-1)$, find d^2y/dx^2

$dy/dx= 2/2x-1$

$d^2y/dx^2= 2(2x-1)^{-2}(2)$

$d^2y/dx^2= -4/(2x-1)^2$

10. Find $\int((3x^2 - x + 2\sqrt{x})/3\sqrt{x})dx$

$= \int(x^{1.5}-\frac{1}{3} x^{0.5}+\frac{2}{3})dx = (x^{2.5}/2.5) - (x^{1.5}/4.5) + (\frac{2}{3})x + c$

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Section B

11. A water tank is filled with water at a constant rate of 20 liters per minute. The tank has a leak at the bottom that drains water at a rate modeled by the function dV/dt = -kV, where V is the volume of water in the tank and k is a constant representing the rate of leakage.

a) Write the equation for the volume of water, V, in the tank as a function of time, t.
The volume of water in the tank at any time, t, is determined by the rate at which water is added to the tank, 20 liters per minute, minus the rate at which water is drained from the tank, -kV. Therefore, the equation for the volume of water in the tank as a function of time is:

$dV/dt = 20 - kV$

b) Determine the constant k from the information given.
From the information given, we know that the leakage rate is modeled by dV/dt = -kV. Since the rate of leakage is a constant, k, we can find its value by rearranging the equation and solving for k. We know that the rate of leakage is equal to the rate at which water is drained from the tank, dV/dt = -kV, so we can substitute the given value of dV/dt = -kV and solve for k:
$k = -dV/dt / V$

c) Use the initial condition V(0) = 0 to find the general solution for the volume of water in the tank.
To find the general solution for the volume of water in the tank, we can use the initial condition that V(0) = 0, since the tank is empty at the start. We can integrate both sides of the equation dV/dt = 20 - kV with respect to t and use the initial condition to find the general solution:

$\int dV = \int(20 - kV)dt$
$V = 20t - (1/2)kV^2 + C$
$V(0) = 0$
$C = 0$
$V = 20t - (1/2)kV^2$

d) Use the general solution to find the volume of water in the tank after 20 minutes.
To find the volume of water in the tank after 20 minutes, we can substitute t = 20 into the general solution for V:
$V(20) = 20(20) - (1/2)k(V(20))^2$

e) Determine the maximum volume of water that the tank can hold.
$V = 20t - (1/2)kV^2 = 0$
$20t = (1/2)kV^2$
$V = \sqrt{(40t/k)}$

In order to find the maximum volume of water the tank can hold, we need to know the value of k. This can be found by measuring the leakage rate or by making assumptions about the tank's structure and the properties of the water. Without this information, we can't find the maximum volume of water that the tank can hold.

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