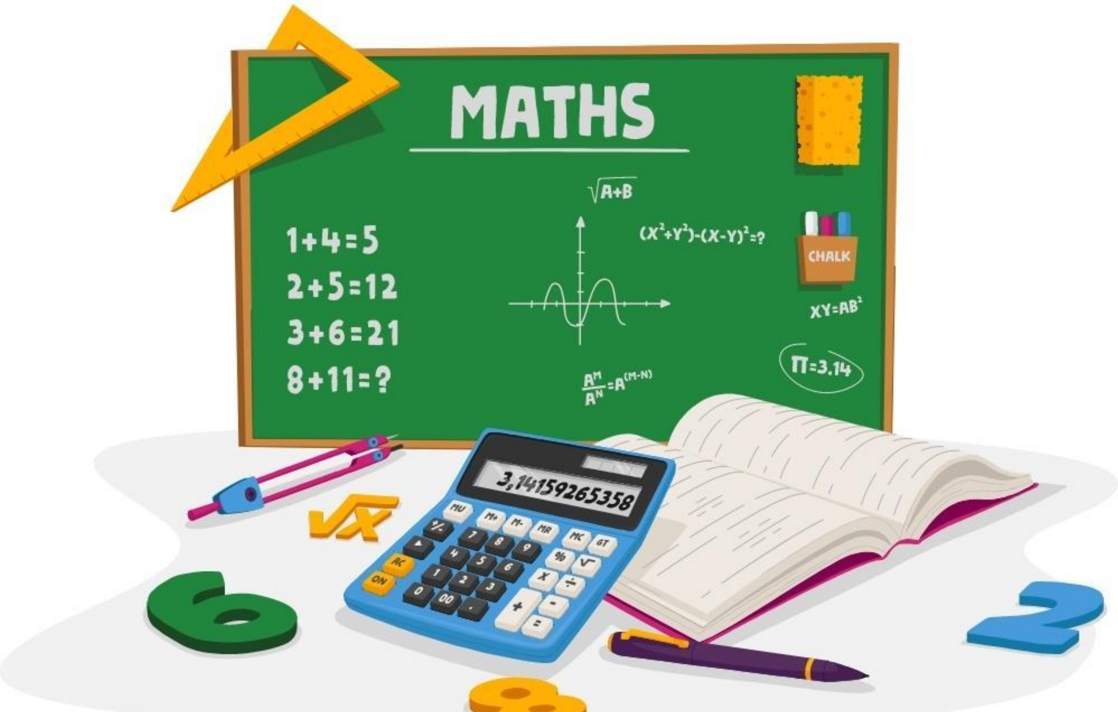
TYCHR

Friend, Philosopher, Guide

IB Maths AA HL Paper 2 Question Bank



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Section A

1. The weight X of a particular animal is normally distributed with μ= 200kg and σ= 15kg. An animal of this population is overweight if it has a weight greater than 230 kg
(a) Find the probability than an animal is overweight.
P (X>230) = 0.02275

(b) We select 2 animals from this population. Find the probabilities that
(i) both animals are overweight
$(0.02275)^2$= 0.000518

(ii) only one animal is overweight.
0.02275)* (1-0.02275)* 2 = 0.0445
(c) We select 7 animals of this population. Find the probability that exactly two of them are overweight.
Y follows B(n,p) with n = 7, p =0.02275
P(Y=2) = 0.00969

2. Consider the independent events A and B. Given that P(B) = 2P(A) and P(A∪B) = 0.52. Find P(B).
Let P(A)= x and P(B)= 2x
0.52 = x + 2x - $2x^2$
x= 0.2
P(B) = 0.4

3. Let f(x) = x^3 - $3x^2$ + 2. Find the point of inflection of the graph of y = f(x).
To find the point of inflection, we need to find the critical points of the function and then determine the concavity. The critical points are found by setting the first derivative of the function to 0 and solving for x.
f'(x) = $3x^2$ - 6x

Setting f'(x) = 0 and solving for x, we find two critical points:
x = 0 and x = 2

To determine the concavity, we need to find the second derivative of the function and evaluate it at the critical points.
f''(x) = 6x - 6
f''(0) = -6 and f''(2) = 6

At x = 0, f''(x) is negative, so the graph is concave down. At x = 2, f''(x) is positive, so the graph is concave up. Since the concavity changes at x = 2, this is the point of inflection. Therefore, the point of inflection of the graph of y = f(x) is (2, f(2)) = (2, -2).

4. A farmer has 600 meters of fencing and wants to enclose a rectangular area for his sheep. He wants the enclosed area to be as large as possible and has a stream running along one side of the rectangle that will serve as the fourth side.

Formulate the problem as a calculus problem and find the maximum area.

We can formulate this problem as an optimization problem using calculus. Let x be the length of the rectangle and y be the width. We know that the total fencing available is 600 meters and that the rectangle has two sides of length x and two sides of length y.
Therefore, the expression for the total fencing used is $2x + 2y = 600$.

The area of the rectangle is given by $A = xy$. We want to maximize this area subject to the constraint $2x + 2y = 600$.

We can start by isolating y in the constraint equation: $y = -x + 300$

Now we can substitute this expression for y into the area expression and get $A = x(-x + 300) = -x^2 + 300x$
We can find the critical points by taking the derivative of the area expression with respect to x and setting it equal to 0:
$A'(x) = -2x + 300 = 0$
$x = 150$

Now we need to check if this critical point is a maximum or minimum. We can do this by taking the second derivative of the area expression and evaluating it at the critical point:
$A''(x) = -2$

Since the second derivative is negative, the critical point $x = 150$ is a maximum. We can substitute this value of x back into the constraint equation to find the corresponding value of y:
$y = -x + 300 = -150 + 300 = 150$

Therefore, the maximum area is 150*150 = 22500 square meters, and the dimensions of the rectangle are 150m by 150m.

5. Consider the following data:

x	1	2	3	4
y	2	3	7	8

(a) Find the mean and the variance for the values of x.
Mean= 2.5, Variance= $1.11803^2 = 1.25$

(b)Find the mean and the variance for the values of y.
Mean= 5, Variance= $2.545951^2 = 6.5$

(c)Find the correlation coefficient r.
0.965

(d)Describe the relation between x and y.
Strong positive

6. Consider the independent events A and B. Given that P(B) = 2P(A) and P(A∪B) = 0.52. Find P(B).
Let P(A)= x and P(B)= 2x
$0.52 = x + 2x - 2x^2$
x= 0.2
P(B) = 0.4

7. Let y= sin(kx) - kxcos(kx), find dy/dx.
dy/dx= kcos(kx) - k(cos(kx) + x(-ksin(kx)))
$= k\cos(kx) - k\cos(kx) + k^2x\sin(kx)$
$= k^2x\sin(kx)$

8. Let $f'(x) = 2e^{-x} + 10\sin 5x + 1$. Find f(x), given that the curve of this function passes through the point A(0,5).
$f(x) = -2e^{-x} - 2\cos 5x + x + c$
f(0) = 5
c= 9
$f(x) = -2e^{-x} - 2\cos 5x + x + 9$

9. A car rental company charges a flat rate of $20 per day plus $0.25 per mile driven. A customer rented a car for x days and drove it a total distance of y miles.
Formulate the cost function C(x,y) of the rental and use it to find the total cost of renting the car for 5 days and driving it 250 miles.

The cost function C(x,y) of the rental can be formulated using the information given in the problem. We know that the flat rate for renting the car is $20 per day and the cost for the distance driven is $0.25 per mile. Therefore, we can write the cost function as
C(x,y) = 20x + 0.25y

To find the total cost of renting the car for 5 days and driving it 250 miles, we substitute x = 5 and y = 250 into the cost function and evaluate:
C(5,250) = 20(5) + 0.25(250) = 100 + 62.5 = $162.50

Therefore, the total cost of renting the car for 5 days and driving it 250 miles is $162.50.

Section B

10. A company has developed a new product and wants to determine the price that will maximize its profit. The company's cost, C, in dollars, to produce x units of the product is modelled by the function $C(x) = 2000 + 2x^2 + 3x$. The revenue, R, in dollars, from selling x units of the product is modelled by the function $R(x) = px$, where p is the unit price in dollars.

a) Write the profit function P(x,p).

The profit function, P(x,p), is the difference between the revenue, R(x), and the cost, C(x).
Therefore, $P(x,p) = R(x) - C(x) = px - (2000 + 2x^2 + 3x)$

b) Determine the number of units, x, that must be sold to break even.

To determine the number of units that must be sold to break even, we need to find the number of units x that make P(x,p) = 0. So, we set P(x,p) = 0 and solve for x:

$px - (2000 + 2x^2 + 3x) = 0$

$px = 2000 + 2x^2 + 3x$

$x = (p - 3)/(2p)$

c) Determine the unit price, p, that will maximize the profit.

To determine the unit price that will maximize the profit, we need to find the value of p that maximizes P(x,p). We can do this by taking the derivative of P(x,p) with respect to p and setting it equal to zero.

$dP/dp = x - (3p - 6x)/p^2 = 0$

$x = (3p - 6x)/p$

d) Determine the maximum profit that can be made.

To determine the maximum profit that can be made, we substitute the value of p found in Part 3 back into the profit function P(x,p) and evaluate at x.

$P(x,p) = px - (2000 + 2x^2 + 3x) = \sqrt{(6x/x + 3)}x - (2000 + 2x^2 + 3x)$

e) Discuss the effect of changes in the cost function on the profit function.

Changes in the cost function will affect the profit function, as it is a part of the cost function which is subtracted from the revenue function to find the profit.

If the cost of production increases, the profit will decrease as the difference between revenue and cost decreases. Conversely, if the cost of production decreases, the profit will increase as the difference between revenue and cost increases.

However, the effect on the profit function will depend on the specifics of the change in the cost function.

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