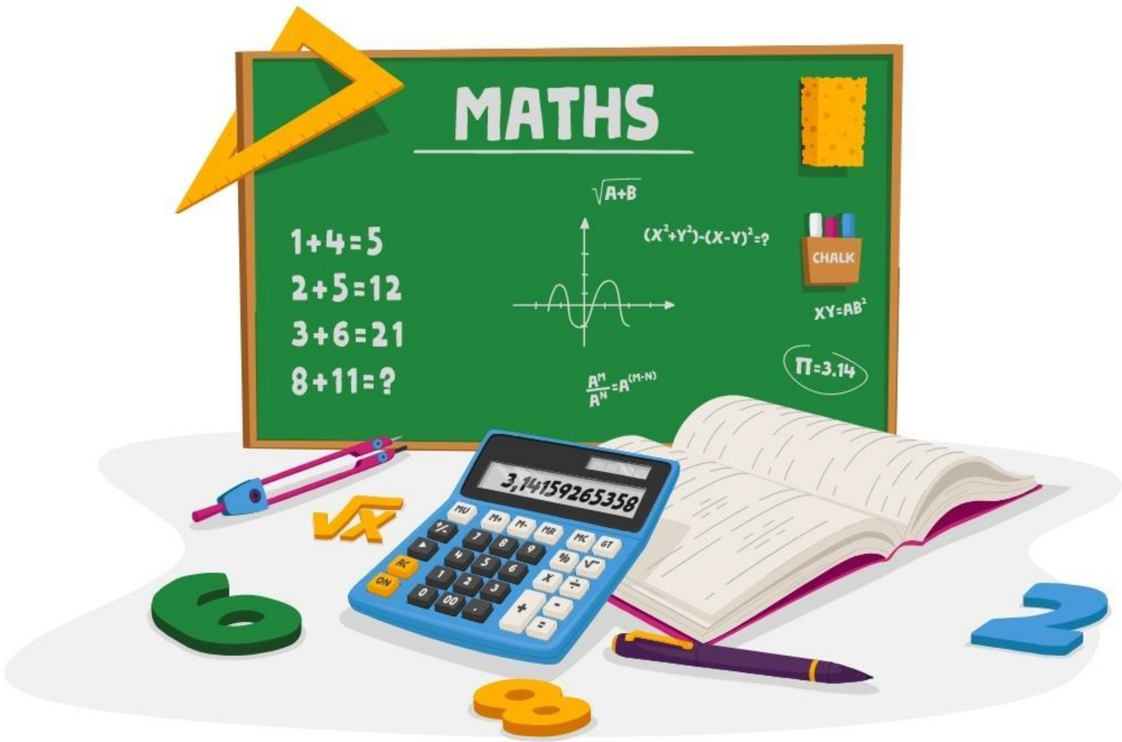
IB Maths AA HL Paper 3 Question Bank



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1- The family of curves $y = f(x) = (ax^3 + bx^2 + cx + d)^n$, where a, b, c, d, and n are constants, is given.
a) Determine the following:
(i) the domain and range of the function y = f(x) for any non-negative values of n.
The domain of the function y = f(x) is all real numbers because the polynomial is defined for all real values of x. The range of the function y = f(x) is all non-negative real numbers because the power of the polynomial is non-negative.

(ii) Determine the x-intercepts of the function y = f(x) for any non-negative values of n.
To find the x-intercepts of the function, we set y = 0 and solve for x. Therefore, we need to find the roots of the polynomial $ax^3 + bx^2 + cx + d = 0$. We can use the factor theorem or the synthetic division to find the roots of the polynomial.

(iii) Determine the y-intercept of the function y = f(x) for any non-negative values of n.
The y-intercept of the function y = f(x) is $(0,d^n)$ since the function is equal to d^n when x = 0.

b) For n = 2, determine the inflection point of the function y = f(x).
For n = 2, the function becomes $y = (ax^3 + bx^2 + cx + d)^2$. To find the inflection point, we need to find the second derivative of the function, which is:
$f''(x) = 6a^{2x} + 2b$

Now we need to find the x values where the second derivative is equal to zero.
$6a^{2x} + 2b = 0$
$x = -b/(3a^2)$

c) Next, Consider the curve, $y^2 = 2x^3 + x$

(i) Find the derivative of $y^2 = 2x^3 + x$
$y^2 = 2x^3 + x$
$y = \sqrt{(2x^3 + x)}$
Next, we can take the derivative of this expression using the chain rule. The chain rule states that if we have a function u(x) = f(g(x)), then the derivative of u(x) with respect to x is given by:
du/dx = df/dg * dg/dx

In this case, we have $u(x) = sqrt(2x^3 + x)$ and we have to find the derivative of this function with respect to x
$du/dx = (1/2)(2x^3 + x)^{(-1/2)}(6x^2 + 1) = (3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$

So the derivative of $y = sqrt(2x^3 + x)$ with respect to x is $(3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$

(ii) Hence, find the local minima or maxima of $y^2 = 2x^3 + x$
To find the local maxima or minima of $y^2 = 2x^3 + x$, we need to find the critical points of the function $y = \sqrt{(2x^3 + x)}$.

A critical point of a function is a point where the derivative of the function is equal to zero or does not exist.

First, we need to find the derivative of $y = \sqrt{(2x^3 + x)}$ with respect to x:
$y' = (3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)})$

Next, we need to set y' equal to zero and solve for x:
$(3x(2x^2 + 1))/(2\sqrt{(2x^3 + x)}) = 0$
$3x(2x^2 + 1) = 0$
$x = 0$

Now, we need to check if x = 0 is a local maximum or minimum. To do this, we need to find the second derivative of $y = \sqrt{(2x^3 + x)}$ with respect to x:
$y'' = (6x(2x^2 + 1) - 3(2x^2 + 1)(2x))/ (2\sqrt{(2x^3 + x)})^2$
We need to evaluate y'' at x = 0, we get
$y''(0) = -3/2$, which is negative and that means that x = 0 is a local minimum.

So the local minima of $y^2 = 2x^3 + x$ is (0, 0)

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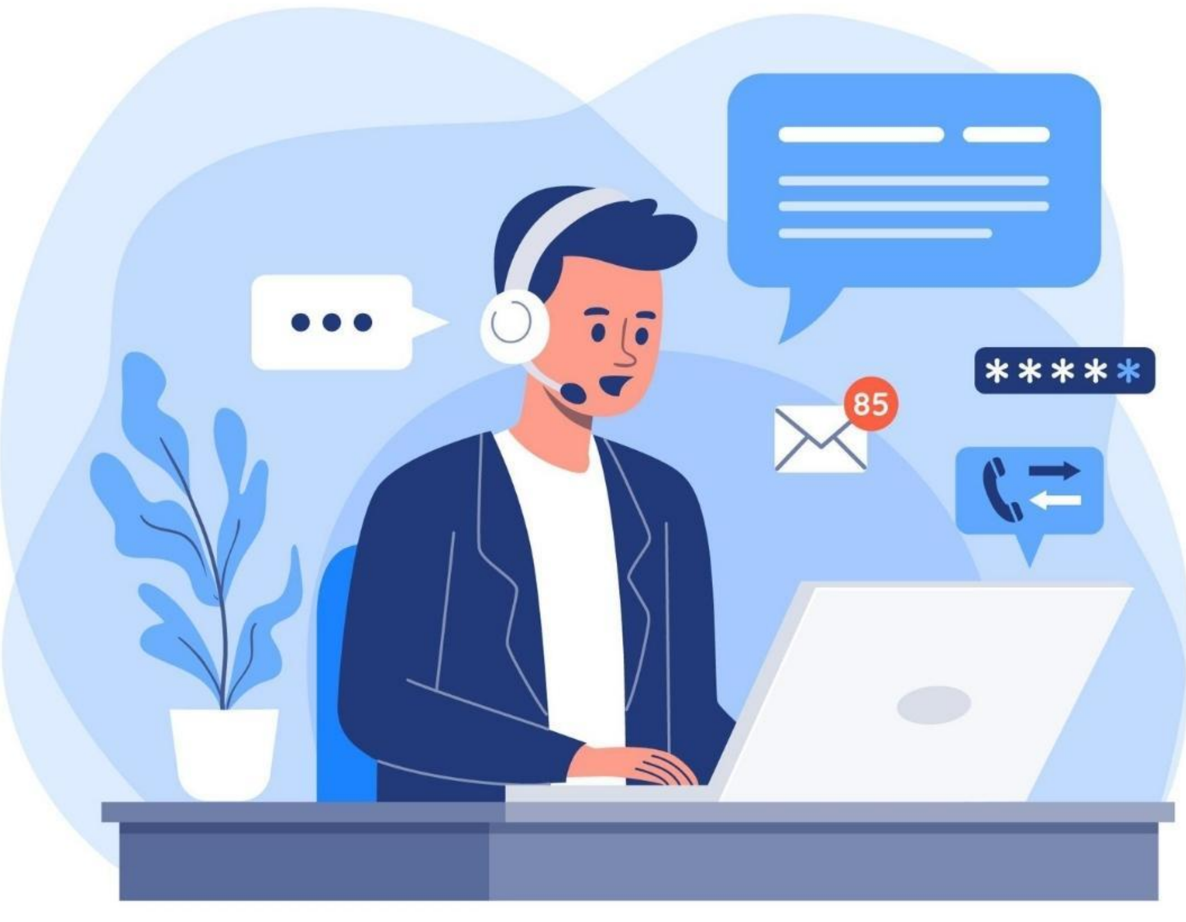
Friend, Philosopher, Guide



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