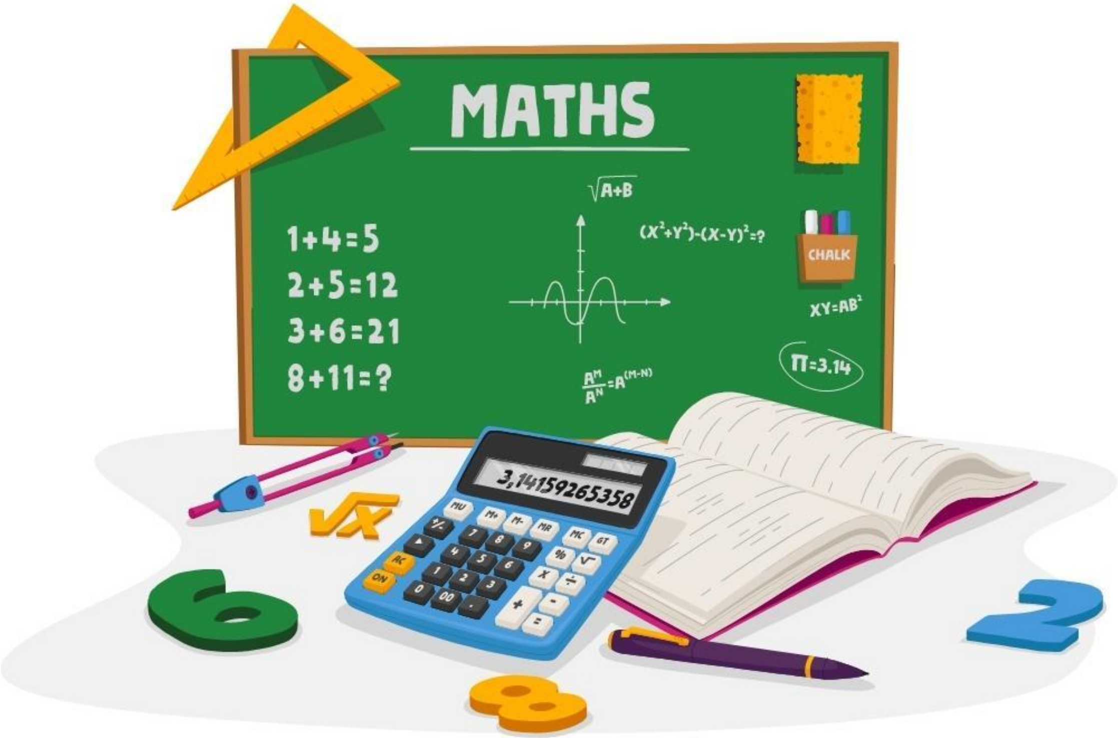
IB Maths AA SL Paper 1 Question Bank



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Section A

<u>1. A ball is thrown vertically upwards from the top of a building 80 meters tall. The height of the ball above the ground, in meters, after t seconds is given by the function h(t) = 80 + 20t - 5t^2.</u>

a) Write down the equation of the motion of the ball in the form of h(t) = v_0t + 1/2at^2

The equation of motion is already given in the form of h(t) = v_0t + 1/2at^2 with v_0 = 20 m/s and a = -5 m/s^2

b) At what time will the ball reach its maximum height?

To find the time at which the ball reaches its maximum height, we need to find the time at which the ball's velocity is zero. The ball's velocity is given by the first derivative of the height function, h'(t) = 20 - 10t. Setting this equal to zero and solving for t gives

0 = 20 - 10t, t = 2s

so, after 2s the velocity will be zero and thus the ball has reached its max height

c) How high is the maximum height?

To find the maximum height, we substitute the value of t = 2 into the height function:

$h(2) = 80 + 20(2) - 5(2)^2 = 80 + 40 - 20 = 100$m

so the maximum height of the ball is 100 meters.

<u>2. The function f(x) = 3x^2 - x^3 is defined on the interval [-1, 1].</u>

a) Find the x-coordinates of any stationary points of f(x)

To find the x-coordinates of any stationary points of f(x), we take the derivative of the function and set it equal to zero:

$f'(x) = 6x - 3x^2 = 3x(2-x) = 0$

Solving for x, we get:

x = 0 or x = 2

Thus, the x-coordinate of the stationary points of f(x) is x = 0 and x = 2

b) Determine the nature of the stationary points found in part (a).

To determine the nature of the stationary points, we need to take the second derivative of the function and test it at the x-coordinates of the stationary points:

$f''(x) = 6 - 6x$

When x = 0, f''(x) = 6 >0 which indicate a Minimum Point

When x = 2, f''(x) = -6 < 0 which indicate a Maximum Point

c) Find the y-coordinate of the stationary point found in part (a)

To find the y-coordinate of the stationary point found in part (a), we substitute the value of x = 0 into the function:

$f(0) = 3(0)^2 - (0)^3 = 0$

So, the y-coordinate of the stationary point is (0,0)

So, this is a Minimum Point and at (0,0)

3. Let $f(x) = x^3 + 2x^2 - 5$. Find the equation of the line which is tangent to the graph of $y = f(x)$ at the point (1, -2).

The equation of the tangent line at a point (a, f(a)) on the graph of $y = f(x)$ is given by
$y = f(a) + f'(a)(x-a)$, where $f'(a)$ is the derivative of $f(x)$ with respect to x evaluated at $x = a$.

So, to find the equation of the tangent line at (1, -2), we first need to find $f'(x)$. We can do this using the power rule and the sum rule of derivatives:
$f'(x) = 3x^2 + 4x$

Now we can substitute $a = 1$ into $f'(a)$ to get $f'(1) = 3(1)^2 + 4(1) = 7$
Therefore, the equation of the tangent line at (1, -2) is:
$y = -2 + 7(x - 1)$
$y = -2 + 7x - 7$
$y = 7x - 9$

4. Consider the quadratic $4x^2-120x+800$

(a)Solve the following
(i) Find the roots.
x=10, x=20

(ii) Hence express the quadratic in the form y= a(x-x1)(x-x2)
y= 4(x-10)(x-20)

(b) Solve the following
(i) Find the coordinates of the vertex.
(15,-100)

(ii) Hence express the quadratic in the form $y= a(x-h)^2+k$
$y= 4(x-15)^2-100$

(iii) Write down the equation of the axis of symmetry
x=15

(iv) Write down the minimum value of y
Ymin= -100

(c) Write down the y-intercept of the quadratic
y=800

5. Solve the equations

a) logx + log(x+1) = log6

Using property: log(A) +log(B) = log(AB)

x(x+1)=6

x^2+x-6=0

x=2

b) logx + log(x+3) = 10

Using property: log(A) +log(B) = log(AB)

x(x+3)=10

x^2+3x-10=0

x=2

c) log(x+18) - logx = 1

Using property: log(A) -log(B) = log(A/B)

(x+18)/x= 1

6. Solve the equation $\tan^2 x = 3$ for $-\pi \le x \le \pi$

tanx = ±√3

For tanx = √3

x=π/3, x= -2π/3

For tanx = -√3

x=-π/3, x= 2π/3

7. Use the sine rule to find the size of side a.

Using sineA/a = sinB/b

a= (10sin45°)/sin30° = 10√2

8. Let g(x) = 2x sinx

a) Find g'(x)

g'(x) = 2sinx + 2xcosx

b) Find the gradient of the graph where x= π

g'(π) = 2sinπ +2πcosπ = -2π

9. Events E and F are independent, with P(E) = ⅔ and P(E∩F) =⅓ . Calculate
a) P(F)
P(F) = ½

b) P(E∪F)
⅔ +½ -⅓ = ⅚

Section B

10. Let $f(x) = x^4 - 4x^3 + 3x^2 + 2x - 1$.

(a) Find the equation of the tangent line to the graph of y = f(x) at the point (1, -2).
To find the equation of the tangent line at a point (a, f(a)) on the graph of y = f(x), we need to use the following equation: y = f(a) + f'(a)(x - a).

Where f'(a) is the derivative of f(x) evaluated at x = a.

First, we find the derivative of f(x) which is:
$f'(x) = 4x^3 - 12x^2 + 6x + 2$

Now we substitute a = 1 into f'(a) to get $f'(1) = 4(1)^3 - 12(1)^2 + 6(1) + 2 = -6$

Therefore, the equation of the tangent line at (1, -2) is:
y = -2 + (-6)(x - 1)
y = -2 - 6x + 6
y = -6x + 8

(b) Find the equation of the normal to the graph of y = f(x) at the point (1, -2).
To find the equation of the normal line at a point (a, f(a)), we need to use the following equation: y - f(a) = -1/f'(a)(x - a)

We know that f'(1) = -6 from part (a)

Therefore, the equation of the normal line at (1, -2) is:
y + 2 = 1/6(x - 1)
y = 1/6x - 1/3

(c) Find the coordinates of the point of intersection of the tangent line and the normal to the graph of y = f(x) at the point (1, -2).
To find the coordinates of the point of intersection of the tangent line and the normal line, we need to solve the system of equations formed by the equations of the tangent line and normal line.

By substituting the equation of the normal line into the equation of the tangent line, we get:
-6x + 8 = 1/6x - 1/3

Solving this equation for x, we get:
x = 3/2

Now we can substitute this value into either of the equations of the tangent line or normal line to find the y-coordinate.

By substituting x = 3/2 into the equation of the tangent line, we get
y = -6x + 8 = -6(3/2) + 8 = -9/2 + 8 = -1/2

Therefore, the point of intersection of the tangent line and the normal line at (1, -2) is (3/2, -1/2)

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