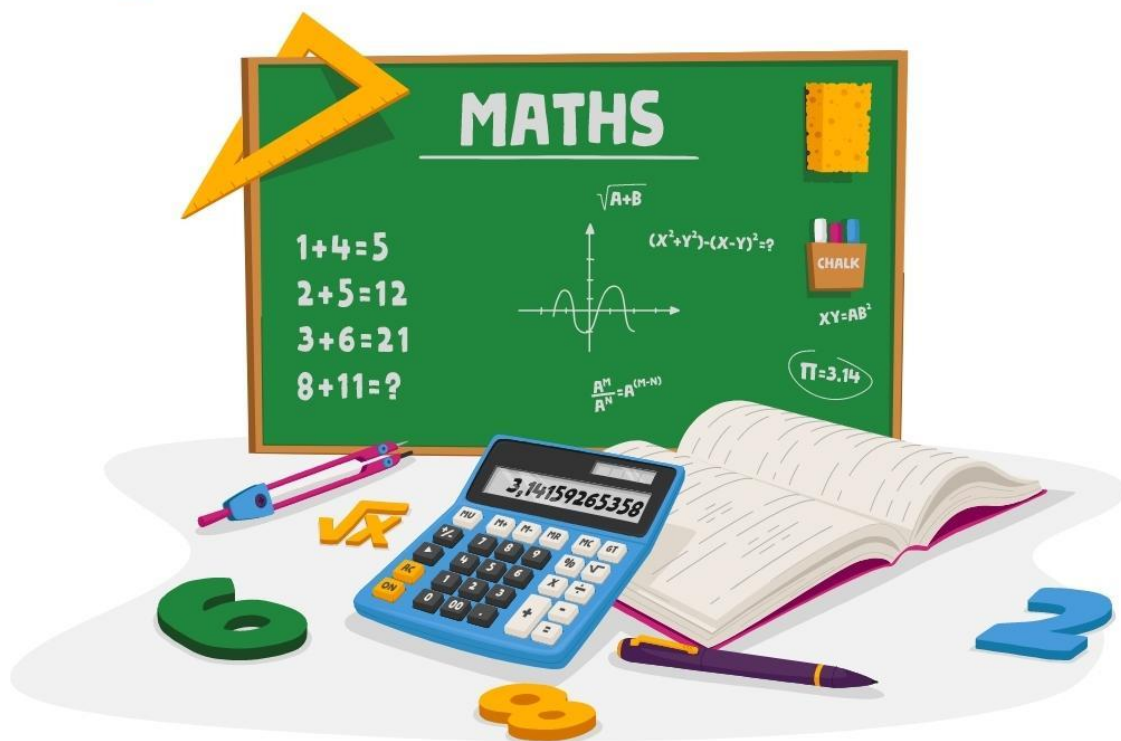


IB Maths AA

SL Paper 2

Question Bank



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Section A

1. A rectangular prism has a volume of 48 cubic meters and a square base with a side length of 2 meters. The height of the prism is h meters.

a) Express the total surface area of the rectangular prism as a function of h.

We know that the volume of a rectangular prism is given by $V = lwh$, where l , w and h are the length, width and height of the prism respectively. In this case, we know that $V = 48 \text{ m}^3$ and the side length of the base is 2 m.

Therefore, the width and length of the base are both 2 m. So, we can express the volume as $V = 2w * 2l * h = 48 \text{ m}^3$

We know the formula for the surface area of a rectangular prism is $2lw + 2lh + 2wh$. Here we know width $w = l = 2\text{m}$ and h is the variable.

So, the surface area A of the rectangular prism as a function of h is:

$$A = 2 * 2 * 2 + 2 * 2 * h = 8 + 4h$$

b) Express the height h of the rectangular prism as a function of the total surface area.

To express the height h as a function of the total surface area, we can use the surface area equation that we found in part a. If we let A be the total surface area, then:

$$A = 8 + 4h$$

Rearranging this equation to solve for h gives us:

$$h = (A-8)/4$$

So, h is a function of the total surface area.

The surface area as a function of h is $A = 8 + 4h$, and the h as a function of the surface area is $h = (A-8)/4$

2. An amount of \$ 10 000 is invested at an annual interest rate of 12%.

(a) Find the value of the investment after 5 years

(i) if the interest rate is compounded yearly;

$$FV = 10000(1 + 12/100)^n$$

$$n = 5, 17623.42$$

(ii) if the interest rate is compounded half-yearly;

$$n = 10, 17908.48$$

(iii) if the interest rate is compounded quarterly;

$$n = 20, 18061.11$$

(iv) if the interest rate is compounded monthly.

$$n = 5 * 12, 18166.97$$

(b) The value of the investment will exceed \$ 20 000 after n full years. Calculate the minimum value of n

(i) if the interest rate is compounded yearly;

$$20000 = 10000(1 + 12/100)^n$$

Solve by GDC

$$n = 6.11, \text{ hence } n = 7$$

(ii) if the interest rate is compounded monthly.

$$20000 = 10000(1 + 12/100)^{12n}$$

Solve by GDC

$$n = 5.81, \text{ hence } n = 6$$

3. A car rental company charges a fixed daily rental fee of \$50, and a variable charge of \$0.15 per kilometre driven.

a) Write an expression for the cost of renting the car for d days, and traveling x kilometers.

The cost of renting the car is made up of two parts: a fixed daily rental fee and a variable charge per kilometer driven. To find the total cost, we need to multiply the number of days the car is rented by the daily rental fee, and add that to the product of the number of kilometers driven and the variable charge per kilometer.

So, the total cost, C, can be expressed as an equation:

$$C = d * 50 + x * 0.15$$

b) The company offers a weekly rental package for \$400, which includes a maximum of 600km of travel. Express the cost of the weekly rental package, C, in terms of the number of kilometres driven, x.

To find the cost of the weekly rental package in terms of the number of kilometers driven, we know that the package includes 600km of travel and cost \$400. So we can create an equation:

$$C = 400 = d * 50 + x * 0.15$$

we know that $d = 7$ for a weekly package and x is 600km

So,

$$C(x) = 7 * 50 + x * 0.15 = 400$$

$$\text{Hence } C(x) = 7 * 50 + 0.15x = 400$$

this is the equation that expresses the cost of the weekly rental package, C, in terms of the number of kilometres driven, x.

4. Triangle ABC is a right-angled triangle, with right angle at B and hypotenuse C.

a) Write down the value of sin B, cos B and tan B in terms of a and b.

In a right-angled triangle, we can use the trigonometric functions sine, cosine, and tangent to relate the lengths of the sides to the measure of the angles.

For a right-angled triangle,

$$\sin B = a/c, \cos B = b/c, \tan B = a/b$$

b) If $a = 5$ and $b = 12$, find the values of $\sin B$, $\cos B$ and $\tan B$.

If we plug in the given values, $a = 5$ and $b = 12$, we can find the values of $\sin B$, $\cos B$ and $\tan B$:

$$\sin B = a/c = 5/c$$

$$\cos B = b/c = 12/c$$

$$\tan B = a/b = 5/12$$

c) Use the result from part (b) to find the value of c .

To find the value of c , we can use the Pythagorean theorem, $c = \sqrt{a^2 + b^2} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$

5. A box with a square base and an open top is to be constructed from a piece of sheet metal which is 25 square meters in area. The box is to have a volume of 40 cubic meters. Find the dimensions of the box.

Let x be the length of one side of the base of the box. Then the area of the base is x^2 and the volume of the box is x^2h , where h is the height of the box. Since the area of the sheet metal is 25 square meters and the volume of the box is 40 cubic meters, we can set up the following equations:

$$x^2 = 25 \text{ (area of base)}$$

$$x^2h = 40 \text{ (volume of the box)}$$

Solving the first equation for x , we find that $x = 5$. So the length of one side of the base of the box is 5 meters. Then, using the second equation, we find that the height of the box is $h = 40/x^2 = 40/25 = 8/5 = 1.6$ meters.

So the dimensions of the box are:

Length: 5 meters

Width: 5 meters

Height: 1.6 meters

6. Solve the following



a) Use the sine rule to find the sine of the angle A

$$\sin A = (10 \sin 30^\circ) / 6$$

$$\sin A = 5/6$$

b) Hence find the two possible values of angle A

$$A = 56.4^\circ \text{ or } A = 124^\circ$$

7. A rectangular prism has a volume of 120 cubic centimetres and a rectangular base with a length 6 cm and a width of 4 cm. Determine the height of the prism.

We know that the volume of a rectangular prism is given by the formula:

$$V = lwh$$

where l is the length, w is the width, and h is the height of the prism.

In this case, we are given that the volume is 120 and the length and width of the base are 6 cm and 4 cm, respectively.

So we have the equation:

$$120 = 6h * 4h$$

To solve for h, we can divide both sides of the equation by $6 * 4$ which will give:

$$120 = 6h * 4h$$

$$120 / 24 = h * h$$

$$5 = h * h$$

$$\text{so } h = \sqrt{5} = 2.236 \text{ cm}$$

8. Consider the independent events A and B. Given that $P(B) = 2P(A)$ and $P(A \cup B) = 0.52$. Find $P(B)$.

Let $P(A) = x$ and $P(B) = 2x$

$$0.52 = x + 2x - 2x^2$$

$$x = 0.2$$

$$P(B) = 0.4$$

9. The weight X of a particular animal is normally distributed with $\mu = 200\text{kg}$ and $\sigma = 15\text{kg}$. An animal of this population is overweight if it has a weight greater than 230 kg

(a) Find the probability that an animal is overweight.

$$P(X > 230) = 0.02275$$

(b) We select 2 animals from this population. Find the probabilities that

(i) both animals are overweight

$$(0.02275)^2 = 0.000518$$

(ii) only one animal is overweight.

$$0.02275 * (1 - 0.02275) * 2 = 0.0445$$

(c) We select 7 animals from this population. Find the probability that exactly two of them are overweight.

Y follows $B(n, p)$ with $n = 7$, $p = 0.02275$

$$P(Y=2) = 0.00969$$

Section B

10. A factory produces electronic components. The probability of a component is defective is 0.05.

a) What is the probability of producing exactly 3 defective components in a batch of 50 components?

This is a binomial probability distribution problem. The probability of producing exactly 3 defective components in a batch of 50 components is:

$$P(X = 3) = (50 \text{ choose } 3) * (0.05)^3 * (1-0.05)^{(50-3)} = 19600 * 0.05^3 * 0.95^{47} \approx 0.09$$

b) What is the probability of producing at most 3 defective components in a batch of 50 components?

To find the probability of producing at most 3 defective components, we can use the cumulative distribution function of the binomial distribution. The probability of producing at most 3 defective components is:

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$$

$$P(X \leq 3) = (50 \text{ choose } 0) * (0.05)^0 * (1-0.05)^{50} + (50 \text{ choose } 1) * (0.05)^1 * (1-0.05)^{49} + (50 \text{ choose } 2) * (0.05)^2 * (1-0.05)^{48} + (50 \text{ choose } 3) * (0.05)^3 * (1-0.05)^{47} \approx 0.37$$

The factory produces 1000 components per day.

c) What is the probability of producing more than 45 defective components in a day?

The expected number of defective components produced per day is:

$$E(X) = \lambda = n * p = 1000 * 0.05 = 50$$

The probability of producing more than 45 defective components in a day is:

$$1 - P(X \leq 45) = 1 - (e^{-50} * (50^{45}) / 45!) \approx 0.02$$

d) What is the probability of producing less than 45 defective components in a day?

To find the probability of producing less than 45 defective components in a day, we can use the Poisson cumulative distribution function. The probability of producing less than 45 defective components in a day is:

$$P(X < 45) = (e^{-50} * (50^{44}) / 44!) \approx 0.98$$



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