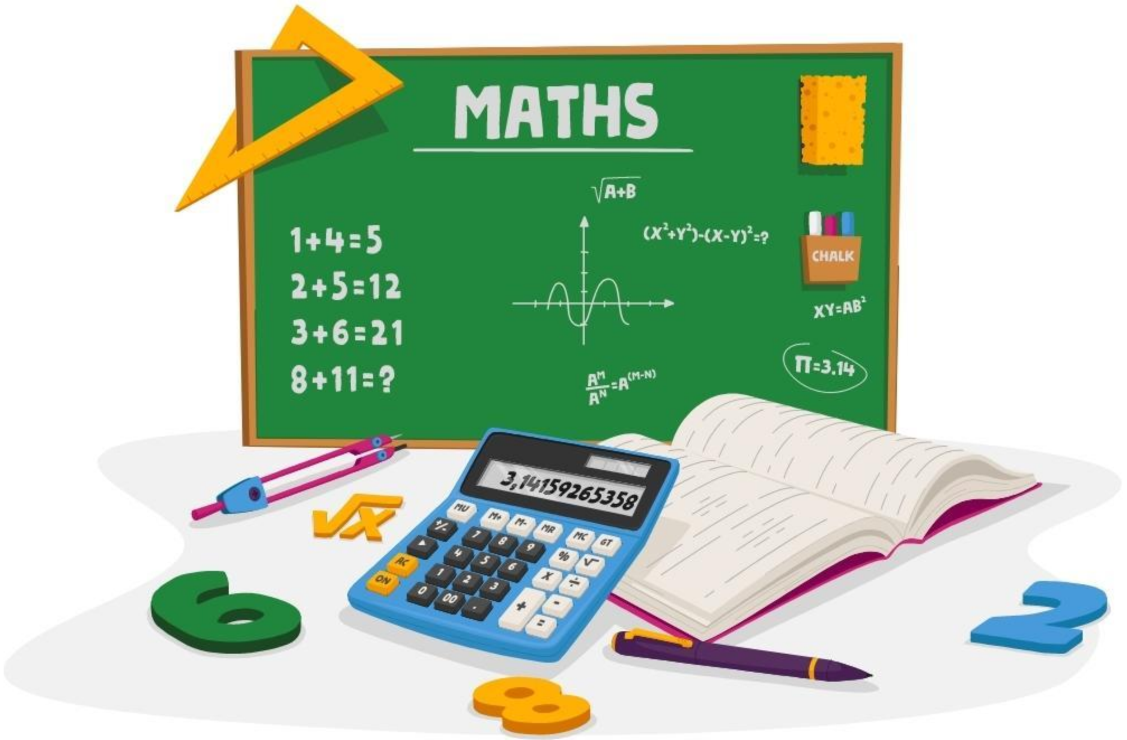
Friend, Philosopher, Guide

IB Maths AI HL Paper 1 Question Bank



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1. Consider the cubic function $f(x) = 2x^3 + ax^2 + bx + c$. Find the value of a, b and c.
(a) if the graph of the function passes through the points (1,0), (–1 ,2),and (0,3).
f(1)= 0, f(1)=2 , f(0)= 3
We finally obtain a=-2, b=-3, c=3

(b) if the graph of the function passes through the points (1,0), (–1,0), and(3,0).
Since we know the 3 roots we know factorization of f(x)
$f(x) = 2(x-1)(x+1)(x-3) = 2(x^2-1)(x-3)$
$2x^3+6x^2-2x+6$
a= -2, b= -2 ,c= 6

2. Solve the equation: $6^{x-1} = 2^{3x+1}$. Express your answer in the form of lna/lnb.
(x-1)ln6 = (3x+1)ln2
Xln6 - ln6 = 3xln2 + ln2
Xln6 - 3xln2 = ln2 +ln6
$X\ln(6/2^3)= \ln 12$
Xln(3/4)= ln12
X =(ln12)/ ln0.75

3. Solve the equation 3sinx = √3cosx
sinx/ cosx = √3/3
tanx=√3/3
x=π/6, x= -5π/3

4. Consider the quadratic $-4x^2+120x-800$

(a) (i) Find the roots.
x=10, x=20

(ii) Hence express the quadratic in the form y= a(x-x1)(x-x2)
y= -4(x-10)(x-20)

(b) (i) Find the coordinates of the vertex.
(15,100)

(ii) Hence express the quadratic in the form $y= a(x-h)^2+k$
$y= -4(x-15)^2+100$

(iii) Write down the equation of the axis of symmetry
x=15

(iv) Write down the maximum value of y
Ymax= -100

(c) Write down the y-intercept of the quadratic
y= -800

5. In a class, 40 students take chemistry only, 30 take physics only, 20 take both chemistry and physics, and 60 take neither.

(a) Find the probability
(i) that a student takes physics given that the student takes chemistry.
P(P I C)= 20/(20+40) = ⅓

(ii) that a student takes physics given that the student does not take chemistry.
P(P I C'') = 30/ (30+60) = ⅓

(b) State whether the events "taking chemistry" and "taking physics" are mutually exclusive, independent, or neither. Justify your answer.
P is independent of C since P(P I C) = P(P) = ⅓

6. Let f(x) = $3x^2$ + 2x - 4 and g(x) = x^3 - 4x.
a) Find the domain and range of f(x) and g(x).
The domain of f(x) is all real numbers because the function is defined for all real values of x. The range of f(x) is also all real numbers, because the function is a polynomial and can take on any real value. Similarly, the domain of g(x) is all real numbers and the range of g(x) is all real numbers.

b) Find the inverse of f(x) and g(x).
To find the inverse of f(x) we need to solve f(x) = y for x.
$3x^2 + 2x - 4 = y$
$x^2 + 2/3\ x - 4/3 = y$
$x = \sqrt{(y + 4/3)} - ⅓$

To find the inverse of g(x) we need to solve g(x) = y for x.
$x^3 - 4x = y$
$x = (y + 4x)^{(1/3)}$

c) Find the x-coordinate of the point of intersection between f(x) and g(x).
To find the x-coordinate of the point of intersection between f(x) and g(x) we need to solve the equation f(x) = g(x)
$3x^2 + 2x - 4 = x^3 - 4x$

We can use the polynomial division or the rational root theorem to find the roots of the equation.
$3x^2 + 2x - 4 = x^3 - 4x$
$x^3 - 3x^2 - 2x + 4 = 0$

By Factorization or synthetic division we can find the roots of the equation which are x = 1, x = -2, x = 2

7. Let f(x) = $x^3 + 2x^2 - 5$. Find the equation of the line which is tangent to the graph of y = f(x) at the point (1, -2).

The equation of the tangent line at a point (a, f(a)) on the graph of y = f(x) is given by
y = f(a) + f'(a)(x-a), where f'(a) is the derivative of f(x) with respect to x evaluated at x = a.

So, to find the equation of the tangent line at (1, -2), we first need to find f'(x). We can do this using the power rule and the sum rule of derivatives:
$f'(x) = 3x^2 + 4x$

Now we can substitute a = 1 into f'(a) to get $f'(1) = 3(1)^2 + 4(1) = 7$
Therefore, the equation of the tangent line at (1, -2) is:
y = -2 + 7(x - 1)
y = -2 + 7x - 7
y = 7x - 9

8. Solve the equation $\tan^2 x = 3$ for $-\pi \leq x \leq \pi$
tanx = ±√3
For tanx = √3
x=π/3, x= -2π/3

For tanx = -√3
x=-π/3, x= 2π/3

9. A box contains 4 red balls, 6 blue balls, and 5 green balls. A ball is randomly selected from the box, and not replaced, and then a second ball is randomly selected from the remaining balls in the box.
a) Find the probability that the first ball selected is red and the second ball selected is blue.
P(Red, Blue) = (4/15) * (6/14) = 24/105

b) Find the probability that the first ball selected is red or the second ball selected is blue.
P(Red or Blue) = P(Red) + P(Blue) - P(Red, Blue) = (4/15) + (6/15) - (24/105) = 40/105

c) Find the probability that the first ball selected is red and the second ball selected is green, given that the first ball selected is red.
P(Green|Red) = P(Red, Green) / P(Red) = (5/15) / (4/15) = 5/4

d) Find the probability that the first ball selected is blue, given that the second ball selected is green.
P(Blue|Green) = P(Green, Blue) / P(Green) = (2/14) / (5/15) = 4/25

e) Are the events "the first ball selected is red" and "the second ball selected is green" independent? Justify your answer.
The events "the first ball selected is red" and "the second ball selected is green" are not independent, because P(Green|Red) is not equal to P(Green).

10. Let $f(x) = x^3 - 3x^2 + 2x + 1$ and $g(x) = x^2 - 4x + 5$.

a) Find the derivative of f(x) using the power rule.

$f'(x) = 3x^2 - 6x + 2$

b) Find the derivative of g(x) using the power rule.

$g'(x) = 2x - 4$

c) Find the points of the intersection of the graphs of y = f(x) and y = g(x).

The point(s) of intersection can be found by solving the equation f(x) = g(x), which gives x = 1 and x = 3.

d) Find the second derivative of f(x) and express it in terms of x.

$f''(x) = 6x - 6$

e) Using the information from parts 1-4, determine the nature of the stationary points of the graph of y = f(x).

Since $f'(x) = 3x^2 - 6x + 2$ is positive for $x < 1$ and $x > 3$, it follows that f(x) is increasing on the intervals (-infinity,1) and (3, infinity) and therefore the stationary points of f(x) are both maxima.

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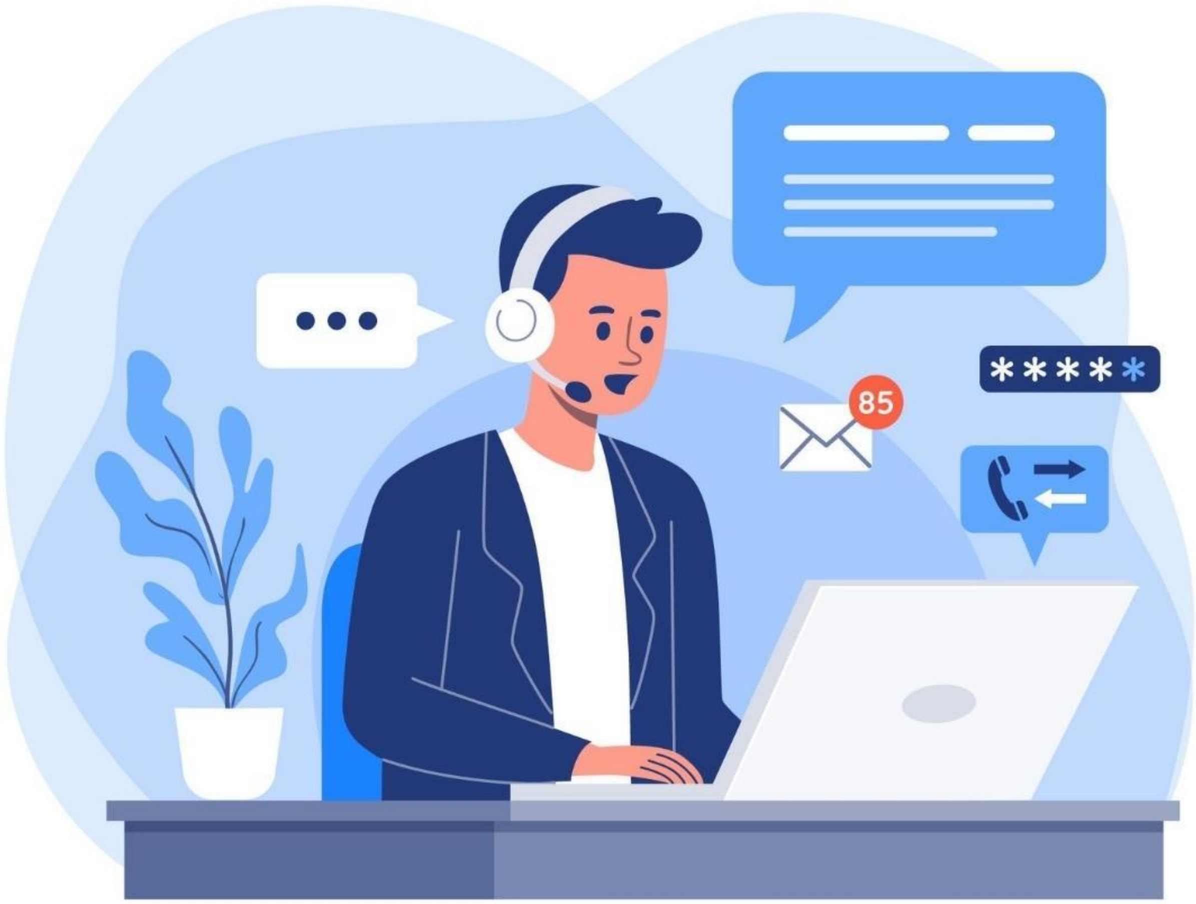

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