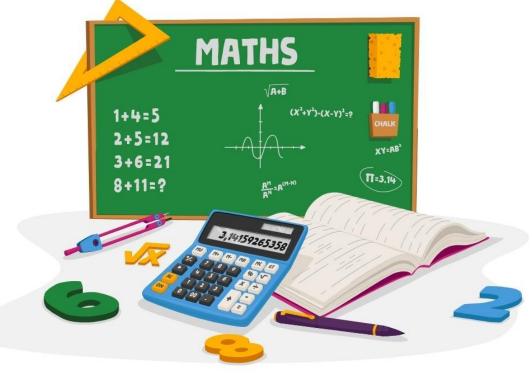


IB Maths Al HL Paper 3 Question Bank



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1- A manufacturing plant produces car parts at a rate of 40 per hour. The production process is Poisson distributed.

a) Determine the following

(i)Find the mean number of car parts produced per hour.

The mean of a Poisson distribution is equal to the rate parameter (λ), which in this case is 40. So the mean number of car parts produced per hour is 40.

(ii) Find the variance of the number of car parts produced per hour.

The variance of a Poisson distribution is also equal to the rate parameter (λ), which in this case is 40. So the variance of the number of car parts produced per hour is also 40.

b) Given that the factory operates for 8 hours a day, what is the probability that the factory will produce exactly 320 car parts in a given day?

To find the probability of producing exactly 320 car parts in a given day, we can use the Poisson probability mass function with $\lambda = 8*40 = 320$, and x = 320. P(320;320) = (e⁻³²⁰) (320³²⁰) / 320! = (2.718⁻³²⁰) (320³²⁰) / 320! = 3.7e-139

A factory produces electronic components. The probability of a component is defective is 0.05.

c) What is the probability of producing exactly 3 defective components in a batch of 50 components?

This is a binomial probability distribution problem. The probability of producing exactly 3 defective components in a batch of 50 components is:

 $P(X = 3) = (50 \text{ choose } 3) * (0.05)^3 * (1-0.05)^{(50-3)} = 19600 * 0.05^3 * 0.95^{47} \approx 0.09$

d)What is the probability of producing at most 3 defective components in a batch of 50 components?

To find the probability of producing at most 3 defective components, we can use the cumulative distribution function of the binomial distribution. The probability of producing at most 3 defective components is:

 $P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

 $\mathsf{P}(\mathsf{X} <= 3) = (50 \text{ choose } 0) * (0.05)^0 * (1-0.05)^{50} + (50 \text{ choose } 1) * (0.05)^1 * (1-0.05)^{49} + (50 \text{ choose } 2) * (0.05)^2 * (1-0.05)^{48} + (50 \text{ choose } 3) * (0.05)^3 * (1-0.05)^{47} \approx 0.37$

The factory produces 1000 components per day.

e) What is the probability of producing more than 45 defective components in a day? The expected number of defective components produced per day is: $E(X) = \lambda = n * p = 1000 * 0.05 = 50$

The probability of producing more than 45 defective components in a day is:

1 - P(X <= 45) = 1 - $(e^{(-50)} * (50^{45}) / 45!) \approx 0.02$





f) What is the probability of producing less than 45 defective components in a day? To find the probability of producing less than 45 defective components in a day, we can use the Poisson cumulative distribution function. The probability of producing less than 45 defective components in a day is:

 $\mathsf{P}(\mathsf{X} < 45) = (\mathrm{e}^{(-50)} * (50^{44}) \, / \, 44!) \approx 0.98$

TYCHR Friend, Philosopher, Guide

