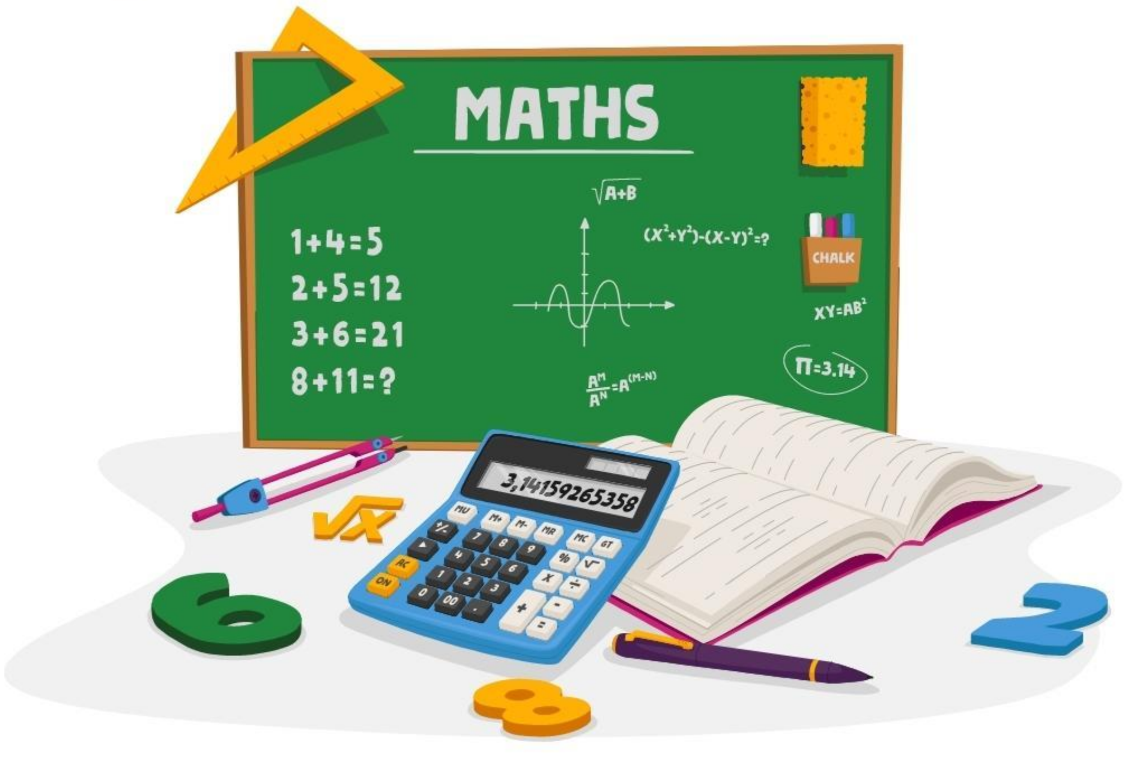
TYCHR

Friend, Philosopher, Guide

IB Maths AI SL Paper 2 Question Bank



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1. Let f(x) = $x^3 - 2x^2 + 3x - 1$ and g(x) = $x^2 + 1$.

(a) Find the composite function (f o g)(x)

To find the composite function (f o g)(x), we substitute g(x) into f(x) to get:

(f o g)(x) = f(g(x)) = $(x^2 + 1)^3 - 2(x^2 + 1)^2 + 3(x^2 + 1) - 1$

(b) Find the inverse function of (f o g)(x)

o find the inverse function of (f o g)(x), we have to find the inverse function of f(x) and g(x) first, and then compose them:

$f^{-1}(x) = (x + 1)^{1/3} + 1/3$ and $g^{-1}(x) = \sqrt{(x - 1)}$

So $(f \circ g)^{-1}(x) = g^{-1}(f^{-1}(x)) = \sqrt{((x + 1)^{1/3} + 1/3 - 1)}$

(c) Find the domain and range of (f o g)(x)

To find the domain and range of (f o g)(x), we need to consider the domain and range of both f(x) and g(x). The domain of g(x) is the set of real numbers, and the range of g(x) is the set of real numbers greater than or equal to 1. The domain of f(x) is the set of real numbers, and the range of f(x) is the set of real numbers.

So the domain of (f o g)(x) is the set of real numbers, and the range of (f o g)(x) is the set of real numbers greater than or equal to 1.

(d) Find the points of discontinuity of (f o g)(x)

To find the points of discontinuity of (f o g)(x), we first need to find the points of discontinuity of both f(x) and g(x).

g(x) has no points of discontinuity.

f(x) has a point of discontinuity at x = 1 as the function is not defined for x=1.

So (f o g)(x) also has a point of discontinuity at x=1.

2. Let f(x) = $e^x - x^2$ and g(x) = $x^3 - 3x^2 + 2x + 1$

a) Find the derivative of f(x) using the chain rule and the product rule.

$f'(x) = e^x - 2x$

b) Find the derivative of g(x) using the power rule

$g'(x) = 3x^2 - 6x + 2$

c) Find the point(s) of the intersection of the graphs of y = f(x) and y = g(x) and state whether they are minima or maxima.

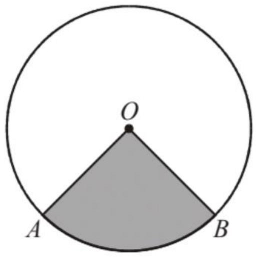
The point(s) of intersection can be found by solving the equation f(x) = g(x), which gives x = 0.5 and x = -0.5. Since the second derivative of f(x) at those points is positive, it follows that the points of intersection are minima.

d) Find the second derivative of f(x) and express it in terms of x.

$f''(x) = e^x - 2$

e) Using the information from parts 1-4, determine the concavity of the graph of y = f(x) and state the point(s) of inflection.
The second derivative of f(x) is positive for $x < -0.5$ and $x > 0.5$, it follows that f(x) is concave up on those intervals, and therefore the points of inflection are $x = -0.5$ and $x = 0.5$.

3. O is the centre of the circle which has a radius of 5.4 cm.



Area of the shaded sector OAB is 21.6 cm^2. Find the length of the minor arc AB
$\frac{1}{2}(5.4)^2\theta = 21.6$
$\theta = 1.481$ radians
$AB = r\theta = 5.4*1.481 = 8cm$

4. In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm. Calculate
a) The size of PQR
$5^2 = 4^2 + 6^2 - 2(4)(6)(\cos Q)$
PQR = 55.8°

b)The area of the triangle PQR
$\frac{1}{2}(4)(6)(\sin 55.8) = 9.92cm^2$

5. The random variable X follows a normal distribution with μ= 100 and σ= 20.
(a) Given that $P(X < a) = 0.8$ find the value of a.
(use tail left, area 0.8) a= 116.8

(b) Given that $P(X > b) = 0.3$ find the value of b.
(use tail right, area 0.3) b=110.5

(c) Find Q1and Q3.
(Use tail central, area 0.5) Q1=86.5 and Q3= 113.5

6. Let $f(x) = 3x^2 + 2x - 4$ and $g(x) = x^3 - 4x$.
a) Find the domain and range of f(x) and g(x).
The domain of f(x) is all real numbers because the function is defined for all real values of x. The range of f(x) is also all real numbers, because the function is a polynomial and can take on any real value. Similarly, the domain of g(x) is all real numbers and the range of g(x) is all real numbers.

b) Find the inverse of f(x) and g(x).
To find the inverse of f(x) we need to solve f(x) = y for x.
$3x^2 + 2x - 4 = y$
$x^2 + 2/3\ x - 4/3 = y$
$x = \sqrt{(y + 4/3)} - \frac{1}{3}$

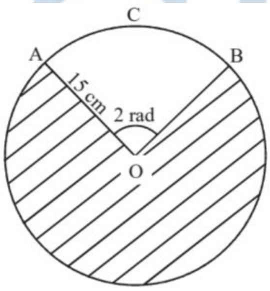
To find the inverse of g(x) we need to solve g(x) = y for x.
$x^3 - 4x = y$
$x = (y + 4x)^{(1/3)}$

c) Find the x-coordinate of the point of intersection between f(x) and g(x).
To find the x-coordinate of the point of intersection between f(x) and g(x) we need to solve the equation f(x) = g(x)
$3x^2 + 2x - 4 = x^3 - 4x$

We can use the polynomial division or the rational root theorem to find the roots of the equation.
$3x^2 + 2x - 4 = x^3 - 4x$
$x^3 - 3x^2 - 2x + 4 = 0$

By Factorization or synthetic division we can find the roots of the equation which are x = 1, x = -2, x = 2

7. The following diagram shows a circle of centre O, and a radius of 15 cm. The arc ACB subtends an angle of 2 radians at the centre O.



a) Find the length of arc ACB.
ACB = 2 × OA= 30cm

b) Find the area of the shaded region.
AOB = 2π - 2
Area = $\frac{1}{2}(r)^2\theta$
$= \frac{1}{2}(15)^2(2\pi - 2)$
$= 482\text{cm}^2$

8. (a) The population of village A increases by 8% every year. If the population today is 1000 people find
$FV = 1000(1+(8/100))^n = 1000(1.08)^n$

(i) the population after 5 years;
n=5, hence 1469

(ii) the population 5 years ago;
n=-5, hence 681

(iii) the number of full years after the population will exceed 2000.
Solve for n : $1000(1.08)^n = 2000$, n=9.0064 so n=10

(b) The population of village B decreases by 8% every year. If the population today is 1000 people find
FV= $1000(1-(8/100))^n = 1000(0.92)^n$

(i) the population after 5 years;
n=5, hence 659

(ii) the population 5 years ago;
n=-5, 1517

(iii) the number of full years after the population will fall under 500
Solve for n : $1000(0.92)^n=500$, n=8.31 so n=9

9. Consider the geometric sequence 10, 5, 2.5, 1.25, …

(a) Write down the first term u1and the common ratio r.
u1= 10, r= 0.5

(b) Find the 10th term of the sequence.
0.0195

(c) Find the sum of the first 10 terms.
19.98

(d) Express the general term un in terms of n.
$10*0.5^{(n-1)}$

(e) Hencefind the value of n given that un.03125
n= 6

10. Let A be the matrix
[1 2 3]
[4 5 6]
[7 8 9]

(a) Find the characteristic polynomial of A

To find the characteristic polynomial of A, we find the determinant of the matrix A - λI, where λ is a scalar and I is the identity matrix.

Characteristic polynomial of A = det(A - λI) = det([1-λ 2 3] [4 5-λ 6] [7 8 9-λ]) = (1-λ)((5-λ)(9-λ) - 86) - 24*7 = $\lambda^3 - 6\lambda^2 + 11\lambda - 6$

(b) Find the eigenvalues of A

To find the eigenvalues of A, we find the roots of the characteristic polynomial.

Eigenvalues of A = roots of $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0 = \lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda-3)(\lambda-2)(\lambda-1) = \lambda = 3, 2, 1$

(c) Find the eigenvectors of A corresponding to the eigenvalues found in (b)

To find the eigenvectors of A corresponding to the eigenvalues found in (b), we solve the system of equations (A - λI)v = 0 for each eigenvalue λ.

For λ = 3, we have (A - 3I)v = 0, which gives us the equation (1-3)x + 2y + 3z = 0, 4x + (5-3)y + 6z = 0, 7x + 8y + (9-3)z = 0

This system can be written as [1 -2 3][x] = [0] , [4 -1 6][y] = [0] , [7 8 -6][z] = [0]

Eigenvectors of A corresponding to λ=3 are $[1\ {-2}\ 3]^T$

For λ = 2, we have (A - 2I)v = 0, which gives us the equation (1-2)x + 2y + 3z = 0, 4x + (5-2)y + 6z = 0, 7x + 8y + (9-2)z = 0

This system can be written as [-1 2 3][x] = [0] , [4 3 6][y] = [0] , [7 8 7][z]

Eigenvectors of A corresponding to λ=2 are $[\ {-1}\ 2\ 3]^T$

For λ = 1, we have (A - I)v = 0, which gives us the equation (1-1)x + 2y + 3z = 0, 4x + (5-1)y + 6z = 0, 7x + 8y + (9-1)z = 0

This system can be written as [0 2 3][x] = [0] , [4 4 6][y] = [0] , [7 8 8][z]

Eigenvectors of A corresponding to λ=1 are $[0\ 2\ 3]^T$

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